Lecture 17

Friday, February 19, 2021 1:59 PM

* Prayer
* Spiritual thought
* Answering questions ...

Last time:

$$f(x,y) = 2xy^2 \quad , \quad (x_0,y_0) = (2,1)$$

Rate of change of f at $(2,1)$ in the direction $\langle 3,2\rangle$.

In this problem,

$$u = \frac{\langle 3,2\rangle}{\sqrt{13}} = \langle a,b\rangle$$

$$D_u f(2,1) = a f_x(2,1) + b f_y(2,1)$$

Notice: this is a linear combination of f_x and f_y.

$$= \frac{3}{\sqrt{13}} f_x(2,1) + \frac{2}{\sqrt{13}} f_y(2,1)$$

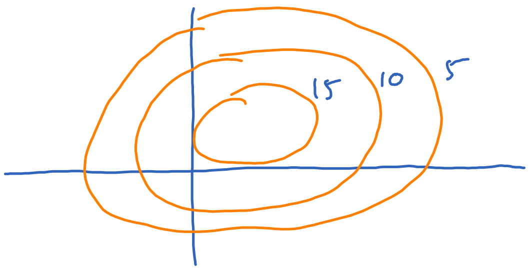
$$= \ ....$$

* Gradient: direction with largest rate of change.

$$u \,/\!/\, \nabla f(x_0,y_0)$$

$$u = \frac{\nabla f(x_0,y_0)}{|\nabla f(x_0,y_0)|}$$

Geometrically, one should be able to tell what direction gives the largest rate of change by looking at the contour map.



Principle: gradient vector is perpendicular to the level curve.

Ex $f(x,y) = xy^3 - x^2$ at $(x_0, y_0) = (1,1)$.

Application:

find tangent plane to the graph $z = x^2 + y^2$ at point $(1,1)$.

$$\underbrace{z = f(x,y)}_{\text{graph of } f} \rightsquigarrow \underbrace{g(x,y,z) = f(x,y) - z}_{\text{is the 0-level set of } g}.$$

Gradient vector: $\nabla g = \langle g_x, g_y, g_z \rangle = \langle f_x, f_y, -1 \rangle$.

$\rightsquigarrow$ Finish the example.

* Local maximum, local minimum

Abs ....., abs .....

* Critical point (stationary)

* Second der. test

$$D = f_{xx} f_{yy} - f_{xy}^2$$

Ex $x^2 + xy + y^2 + y$

$x^4 - 2x^2 + y^3 - 3y$

$(x-y)(1-xy)$